

- The transition point for a boundary layer on a flat plate occurs at a nominal Reynolds number of 5×10^5 based on the free stream velocity and the distance from the leading edge.
- The turbulent boundary layer is characterized by an unsteady flow where the momentum exchange between fluid layers occurs because of the mixing of fluid elements normal to the direction of fluid motion. This effect, known as the Reynolds stress, significantly enhances the momentum exchange and leads to a much higher “effective” shear stress.

Predicting Shear Stress and Shear Force

- The local shear-stress coefficient is defined as

$$c_f = \frac{\tau_0}{\frac{1}{2} \rho U_0^2}$$

where τ_0 is the wall shear stress and U_0 is the free-stream velocity.

- The value for the local shear-stress coefficient on a flat plate depends on the Reynolds number based on the distance from the leading edge.
- The average shear-stress coefficient is

$$C_f = \frac{F_s}{\frac{1}{2} \rho U_0^2 A}$$

where F_s is the force due to shear-stress, or shear force, on a surface with area A .

- The value for the average shear-stress coefficient for a flat plate depends on the nature of the boundary layer as related to the Reynolds number based on the length of the plate in the flow direction.
- The laminar boundary layer near the leading edge and the subsequent turbulent boundary layer contribute to the average shear stress on a flat plate.
- Through leading-edge roughness or other flow disturbance, the boundary layer can be “tripped” at the plate’s leading edge, effecting a turbulent boundary layer over the entire plate.

Effects of Pressure Gradient

- The boundary layer for flow over a curved body is subjected to an external pressure gradient.
- A favorable pressure gradient produces a force in the flow direction and tends to keep the boundary layer stable.
- An adverse pressure gradient decelerates the flow and can lead to boundary layer separation.

REFERENCES

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PROBLEMS

PLUS Problem available in WileyPLUS at instructor’s discretion.

GO Guided Online (GO) Problem, available in WileyPLUS at instructor’s discretion.

Uniform Laminar Flow (\$9.1)

9.1 **PLUS** In which case is the flow caused by a pressure gradient?

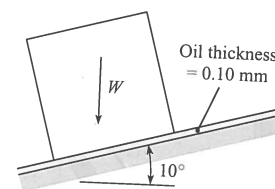
- a. Couette flow
- b. Hele-Shaw flow

9.2 The velocity distribution in a Couette flow is linear if the viscosity is constant. If the moving plate is heated and the viscosity of the liquid is decreased near the hot plate, how will the velocity distribution change? Give a qualitative description and the rationale for your argument.

9.3 Consider the flow of various fluids between two parallel plates.

- a. Assume the fluid is a liquid, its viscosity is constant along the flow direction, and the pressure gradient is linear with distance. How would the pressure gradient differ if the viscosity of the fluid decreased (due to temperature rise) along the flow direction. The density is unchanged. Give a qualitative description of pressure distribution and provide rationale for your answer.
- b. Assume the fluid is a gas flowing between two parallel plates. If there were an increase in temperature due to heat transfer along the flow direction, the gas density would decrease. Assume the viscosity is unaffected. How will the velocity and pressure distribution change from the case with constant density? Sketch the pressure distribution and give the rationale for your result.

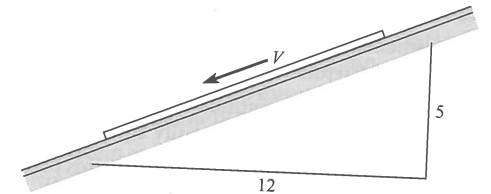
9.4 **PLUS** The cube shown weighing 110 N and measuring 39 cm on a side is allowed to slide down an inclined surface on which there is a film of oil having a viscosity of $10^{-2} \text{ N} \cdot \text{s}/\text{m}^2$. What is the velocity of the block if the oil has a thickness of 0.11 mm?



PROBLEM 9.4

9.5 **PLUS** A board 1 m by 1 m that weighs 180 N slides down an inclined ramp with a velocity of 0.15 m/s. The board is separated from the ramp by a layer of oil 0.05 cm thick. Neglecting the edge effects of the board, calculate the approximate dynamic viscosity μ of the oil.

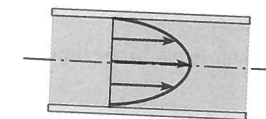
9.6 A board 1 m by 1 m that weighs 30 N slides down an inclined ramp with a velocity of 17 cm/s. The board is separated from the ramp by a layer of oil 0.8 mm thick. Neglecting the edge effects of the board, calculate the approximate dynamic viscosity μ of the oil.



PROBLEMS 9.5, 9.6

9.7 **PLUS** Uniform, steady flow is occurring between horizontal parallel plates as shown.

- a. The flow is Hele-Shaw; therefore, what is causing the fluid to move?
- b. Where is the maximum velocity located?
- c. Where is the maximum shear stress located?
- d. Where is the minimum shear stress located?



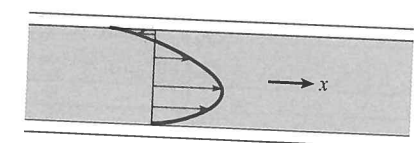
PROBLEM 9.7

9.8 Uniform, steady flow is occurring between horizontal parallel plates as shown.

- a. In a few words, tell what other condition must be present to cause the odd velocity distribution.
- b. Where is the minimum shear stress located?

9.9 **PLUS** Under certain conditions (pressure decreasing in the x -direction, the upper plate fixed, and the lower plate moving to the right in the positive x -direction), the laminar velocity distribution will be as shown. For such conditions, indicate whether each of the following statements is true or false.

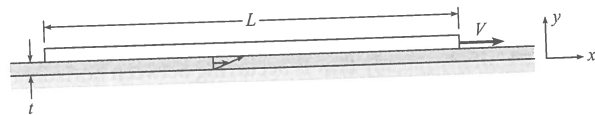
- a. The shear stress midway between the plates is zero.
- b. The minimum shear stress in the liquid occurs next to the moving plate.
- c. The shear stress is greatest where the velocity is the greatest.
- d. The minimum shear stress occurs where the velocity is the greatest.



PROBLEMS 9.8, 9.9

9.10 A flat plate is pulled to the right at a speed of 30 cm/s. Oil with a viscosity of $4 \text{ N} \cdot \text{s}/\text{m}^2$ fills the space between the plate and the solid boundary. The plate is 1 m long ($L = 1 \text{ m}$) by 30 cm wide, and the spacing between the plate and boundary is 2.0 mm.

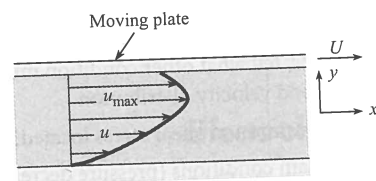
- Express the velocity mathematically in terms of the coordinate system shown.
- By mathematical means, determine whether this flow is rotational or irrotational.
- Determine whether continuity is satisfied, using the differential form of the continuity equation.
- Calculate the force required to produce this plate motion.



PROBLEM 9.10

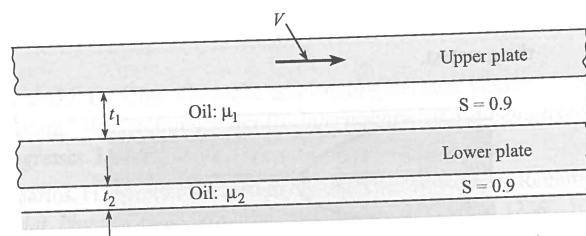
9.11 The velocity distribution that is shown represents laminar flow. Indicate which of the following statements are true.

- The velocity gradient at the boundary is infinitely large.
- The maximum shear stress in the liquid occurs midway between the walls.
- The maximum shear stress in the liquid occurs next to the boundary.
- The flow is irrotational.
- The flow is rotational.



PROBLEM 9.11

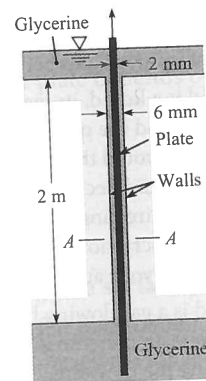
9.12 The upper plate shown is moving to the right with a velocity V , and the lower plate is free to move laterally under the action of the viscous forces applied to it. For steady-state conditions, derive an equation for the velocity of the lower plate. Assume that the area of oil contact is the same for the upper plate, each side of the lower plate, and the fixed boundary.



PROBLEM 9.12

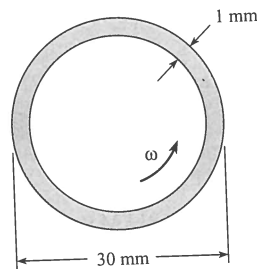
9.13 A circular horizontal disk with a 27 cm diameter has a clearance of 3.0 mm from a horizontal plate. What torque is required to rotate the disk about its center at an angular speed of 31 rad/s when the clearance space contains oil ($\mu = 8 \text{ N} \cdot \text{s}/\text{m}^2$)?

9.14 A plate 2 mm thick and 1 m wide (normal to the page) is pulled between the walls shown in the figure at a speed of 0.40 m/s. Note that the space that is not occupied by the plate is filled with glycerine at a temperature of 20°C . Also, the plate is positioned midway between the walls. Sketch the velocity distribution of the glycerine at section A-A. Neglecting the weight of the plate, estimate the force required to pull the plate at the speed given.



PROBLEM 9.14

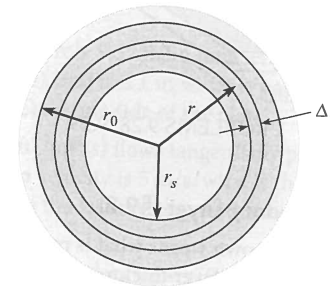
9.15 A bearing uses SAE 30 oil with a viscosity of $0.1 \text{ N} \cdot \text{s}/\text{m}^2$. The bearing is 30 mm in diameter, and the gap between the shaft and the casing is 1 mm. The bearing has a length of 1 cm. The shaft turns at $\omega = 200 \text{ rad/s}$. Assuming that the flow between the shaft and the casing is a Couette flow, find the torque required to turn the bearing.



PROBLEM 9.15

9.16 An important application of viscous flow is found in lubrication theory. Consider a shaft that turns inside a stationary cylinder, with a lubricating fluid in the annular region. By considering a system consisting of an annulus of fluid of radius r and width Δr , and realizing that under steady-state operation the net torque on this ring is zero, show that $d(r^2\tau)/dr = 0$, where τ is the viscous shear stress. For a flow that has a tangential component of velocity only, the shear stress is related to the

velocity by $\tau = \mu r d(V/r)/dr$. Show that the torque per unit length acting on the inner cylinder is given by $T = 4\pi\mu\omega r_s^2/(1 - r_s^2/r_0^2)$, where ω is the angular velocity of the shaft.



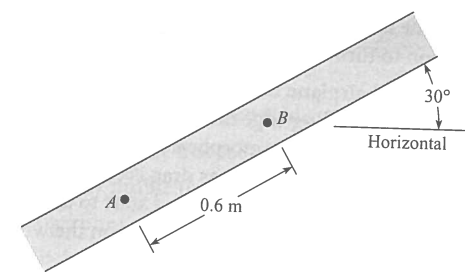
PROBLEM 9.16

9.17 Using the equation developed in Prob. 9.16, find the power necessary to rotate a 2 cm shaft at 60 rad/s if the inside diameter of the casing is 2.2 cm, the bearing is 3 cm long, and SAE 30 oil at 38°C is the lubricating fluid.

9.18 The analysis developed in Prob. 9.16 applies to a device used to measure the viscosity of a fluid. By applying a known torque to the inner cylinder and measuring the angular velocity achieved, one can calculate the viscosity of the fluid. Assume you have a 4 cm inner cylinder and a 4.5 cm outer cylinder. The cylinders are 10 cm long. When a force of 0.6 N is applied to the tangent of the inner cylinder, it rotates at 20 rpm. Calculate the viscosity of the fluid.

9.19 Two horizontal parallel plates are spaced 0.0045 m apart. The pressure decreases at a rate of 3650 Pa/m in the horizontal x -direction in the fluid between the plates. What is the maximum fluid velocity in the x direction? The fluid has a dynamic viscosity of $0.05 \text{ N} \cdot \text{s}/\text{m}^2$ and a specific gravity of 0.80.

9.20 A viscous fluid fills the space between these two plates, and the pressures at A and B are 7 kPa and 5 kPa, respectively. The fluid is not accelerating. If the specific weight of the fluid is $15.7 \text{ kN}/\text{m}^3$, then one must conclude that (a) flow is downward, (b) flow is upward, or (c) there is no flow.



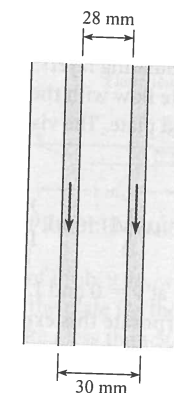
PROBLEM 9.20

9.21 Glycerine at 20°C flows downward between two vertical parallel plates separated by a distance of 0.4 cm. The ends are open, so there is no pressure gradient. Calculate the discharge per unit width, q , in m^2/s .

9.22 Two vertical parallel plates are spaced 3 mm apart. If the pressure decreases at a rate of 9400 Pa/m in the vertical z -direction in the fluid between the plates, what is the maximum fluid velocity in the z -direction? The fluid has a viscosity of $0.05 \text{ Pa} \cdot \text{s}$ and a specific gravity of 0.80.

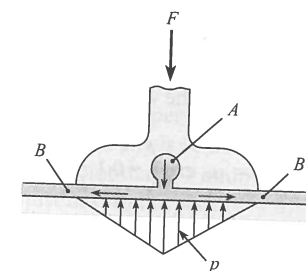
9.23 Two parallel plates are spaced 2 mm apart, and motor oil (SAE 30) with a temperature of 38°C flows at a rate of $0.00025 \text{ m}^3/\text{s}$ per meter of width between the plates. What is the pressure gradient in the direction of flow if the plates are inclined at 60° with the horizontal and if the flow is downward between the plates?

9.24 Glycerin at 20°C flows downward in the annular region between two cylinders. The internal diameter of the outer cylinder is 3 cm, and the external diameter of the inner cylinder is 2.8 cm. The pressure is constant along the flow direction. The flow is laminar. Calculate the discharge. (Hint: The flow between the two cylinders can be treated as the flow between two flat plates.)



PROBLEM 9.24

9.25 One type of bearing that can be used to support very large structures is shown in the accompanying figure. Here fluid under pressure is forced from the bearing midpoint (slot A) to the exterior zone B. Thus a pressure distribution occurs as shown. For this bearing, which is 43 cm wide, what discharge of oil from slot A per meter of length of bearing is required? Assume a 190 kN load per meter of bearing length with a clearance space t between the floor and the bearing surface of 1.5 mm. Assume an oil viscosity of $0.20 \text{ N} \cdot \text{s}/\text{m}^2$. How much oil per hour would have to be pumped per meter of bearing length for the given conditions?



PROBLEM 9.25

9.26 Often in liquid lubrication applications there is a heat generated that is transferred across the lubricating layer. Consider a Couette flow with one wall at a higher temperature than the other. The temperature gradient across the flow affects the fluid viscosity according to the relationship.

$$\mu = \mu_0 \exp\left(-0.1 \frac{y}{L}\right)$$

where μ_0 is the viscosity at $y = 0$ and L is the distance between the walls. Incorporate this expression into the Couette flow equation, integrate and express the shear stress in the form

$$\tau = C \frac{U\mu_0}{L}$$

where C is a constant and U is the velocity of the moving wall. Analyze your answer. Should the shear stress be greater or less than that with uniform viscosity?


9.27 Gases form good insulating layers. Consider an application in which there is a Couette flow with the moving plate at a higher temperature than the fixed plate. The viscosity varies between the plates as

$$\mu = \mu_0 \left(1 + 0.1 \frac{y}{L}\right)^{1/2}$$

where μ_0 is the viscosity at $y = 0$ and L is the distance between the plates. Incorporate this expression into the Couette flow equation, integrate and express the shear stress in the form

$$\tau = C \frac{U\mu_0}{L}$$

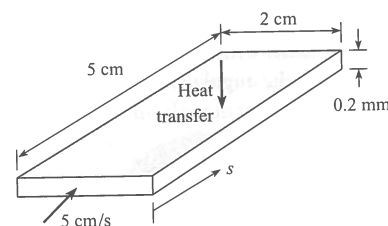
where C is a constant and U is the velocity of the moving plate. Analyze your answer. Should the shear stress be greater or less than that with uniform viscosity?

9.28  An engineer is designing a very thin, horizontal channel for cooling electronic circuitry. The channel is 2 cm wide and 5 cm long. The distance between the plates is 0.2 mm. The average velocity is 5 cm/s. The fluid used has a viscosity of 1.2 cp and a density of 800 kg/m³. Assuming no change in viscosity or density, find the pressure drop in the channel and the power required to move the flow through the channel.

9.29 Consider the channel designed for electronic cooling in Prob. 9.28. Because of the heating, the viscosity will change through the channel. Assume the viscosity varies as

$$\mu = \mu_0 \exp\left(-0.1 \frac{s}{L}\right)$$


where μ_0 is the viscosity at $s = 0$ and L is the length of the channel. Find the percentage change of the pressure drop due to viscosity variation.



PROBLEMS 9.28, 9.29

Describing the Boundary Layer (§9.2)


9.30 a. Explain in your own words what is meant by “boundary layer.” b. Define “boundary layer thickness.”

9.31  Which of the following are features of a laminar boundary layer? (Select all that are correct.)


- Flow is smooth.
- The boundary layer thickness increases in the downstream direction.
- A decreasing boundary layer thickness correlates with decreased shear stress.
- An increasing boundary layer thickness correlates with decreased shear stress.

Laminar Boundary Layer (§9.3)

9.32 Assume the wall adjacent to a liquid laminar boundary is heated and the viscosity of the fluid is lower near the wall and increases the free-stream value at the edge of the boundary layer. How would this variation in viscosity affect the boundary-layer thickness and local shear stress? Give the rationale for your answers.

9.33  A thin plate 2 m long and 1 m wide is submerged and held stationary in a stream of water ($T = 15^\circ\text{C}$) that has a velocity of 1.5 m/s. What is the thickness of the boundary layer on the plate for $Re_x = 500,000$ (assume the boundary layer is still laminar), and at what distance downstream of the leading edge does this Reynolds number occur? What is the shear stress on the plate at this point?

9.34 What is the ratio of the boundary-layer thickness on a smooth, flat plate to the distance from the leading edge just before transition to turbulent flow?


9.35  A model airplane has a wing span of 1 m and a chord (leading edge–trailing edge distance) of 12.5 cm. The model flies in air at 15°C and atmospheric pressure. The wing can be regarded as a flat plate so far as drag is concerned. (a) At what speed will a turbulent boundary layer start to develop on the wing? (b) What will be the total drag force on the wing just before turbulence appears?

9.36 Oil ($\mu = 10^{-2} \text{ N} \cdot \text{s/m}^2$; $\rho = 900 \text{ kg/m}^3$) flows past a plate in a tangential direction so that a boundary layer develops. If the velocity of approach is 4 m/s, then at a section 30 cm downstream of the leading edge the ratio of τ_δ (shear stress at the edge of the boundary layer) to τ_0 (shear stress at the plate surface) is approximately (a) 0, (b) 0.24, (c) 2.4, or (d) 24.


9.37 A liquid ($\rho = 1000 \text{ kg/m}^3$; $\mu = 2 \times 10^{-2} \text{ N} \cdot \text{s/m}^2$; $\nu = 2 \times 10^{-5} \text{ m}^2/\text{s}$) flows tangentially past a flat plate. If the approach velocity is 2 m/s, what is the liquid velocity 1 m downstream from the leading edge of the plate, at 0.8 mm away from the plate?

9.38 The plate of Prob. 9.37 has a total length of 3 m (parallel to the flow direction), and it is 1 m wide. What is the skin friction drag (shear force) on one side of the plate?

9.39 Oil ($\nu = 10^{-4} \text{ m}^2/\text{s}$) flows tangentially past a thin plate. If the free-stream velocity is 5 m/s, what is the velocity 1 m downstream from the leading edge and 3 mm away from the plate?


9.40  Oil ($\nu = 10^{-4} \text{ m}^2/\text{s}$; $S = 0.9$) flows past a plate in a tangential direction so that a boundary layer develops. If the velocity of approach is 0.85 m/s, what is the oil velocity 1.6 m downstream from the leading edge, 10 cm away from the plate?

9.41 A thin plate 0.7 m long and 1.5 m wide is submerged and held stationary in a stream of water ($T = 10^\circ\text{C}$) that has a velocity of 1.5 m/s. What is the thickness of the boundary layer on the plate for $Re_x = 500,000$ (assume the boundary layer is still laminar), and at what distance downstream of the leading edge does this Reynolds number occur? What is the shear stress on the plate on this point?

9.42  A flat plate 1.5 m long and 1.0 m wide is towed in water at 20°C in the direction of its length at a speed of 15 cm/s. Determine the resistance of the plate and the boundary layer thickness at its aft end.


9.43 Transition from a laminar to a turbulent boundary layer occurs between the Reynolds numbers of $Re_x = 10^5$ and $Re_x = 3 \times 10^6$. The thickness of the turbulent boundary layer based on the distance from the leading edge is $\delta = 0.16x/(Re_x)^{1/7}$. Find the ratio of the thickness of the laminar boundary layer at the beginning of transition to the thickness of the turbulent boundary layer at the end of transition.

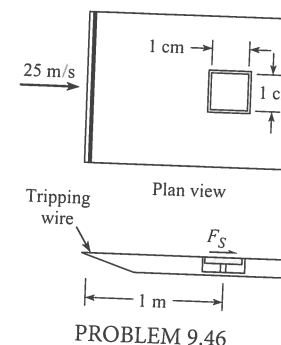
Turbulent Boundary Layer (§9.5)

9.44  Classify each of the following features into one of two categories: laminar boundary layer (L), or turbulent boundary layer (T).

- Flow is smooth
- Three differently shaped velocity distributions in 3 zones
- Velocity profile that follows a power law
- Velocity profile that is a function of \sqrt{Re}
- Logarithmic velocity distribution
- Thickness is inversely related to the 7th root of Re
- Thickness is inversely related to \sqrt{Re}
- Velocity defect region
- Mixing action causes locally unsteady velocities
- Shear stress is a function of a natural log
- Shear stress is a function of \sqrt{Re}


9.45 Assume that a turbulent gas boundary layer was adjacent to a cool wall and the viscosity in the wall region was reduced. How may this affect the features of the boundary layer? Give some rationale for your answers.

9.46  An element for sensing local shear stress is positioned in a flat plate 1 meter from the leading edge. The element simply consists of a small plate, 1 cm \times 1 cm, mounted flush with the wall, and the shear force is measured on the plate. The fluid flowing by the plate is air with a free-stream velocity of $V = 30 \text{ m/s}$, a density of 1.2 kg/m^3 , and a kinematic viscosity of $1.5 \times 10^{-5} \text{ m}^2/\text{s}$. The boundary layer is tripped at the leading edge. What is the magnitude of the force due to shear stress acting on the element?




PROBLEM 9.46

9.47 For the conditions of Prob. 9.46, what is the shearing resistance on one side of the plate for the part of the plate that has a Reynolds number, Re_x , less than 500,000? What is the ratio of the laminar shearing force to the total shearing force on the plate?

9.48  An airplane wing of 2 m chord length (leading edge to trailing edge distance) and 11 m span flies at 200 km/hr in air at 30°C . Assume that the resistance of the wing surfaces is like that of a flat plate.

- What is the friction drag on the wing?
- What power is required to overcome this?
- How much of the chord is laminar?
- What will be the change in drag if a turbulent boundary layer is tripped at the leading edge?

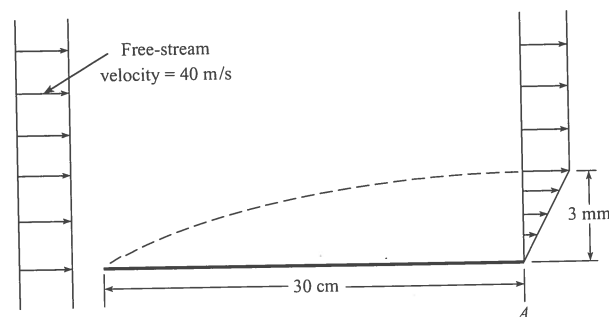
9.49  A turbulent boundary layer exists in the flow of water at 20°C over a flat plate. The local shear stress measured at the surface of the plate is 0.2 N/m^2 . What is the velocity at a point 0.52 cm from the plate surface?

9.50 A liquid flows tangentially past a flat plate. The fluid properties are $\mu = 10^{-5} \text{ N} \cdot \text{s/m}^2$ and $\rho = 1.5 \text{ kg/m}^3$. Find the skin friction drag on the plate per unit width if the plate is 2.5 m long and the approach velocity is 16 m/s. Also, what is the velocity gradient at a point that is 1 m downstream of the leading edge and just next to the plate ($y = 0$)?

9.51 For the hypothetical boundary layer on the flat plate shown, what is the shear-stress on the plate at the downstream end (point A)? Here $\rho = 1.2 \text{ kg/m}^3$ and $\mu = 1.8 \times 10^{-5} \text{ N} \cdot \text{s/m}^2$.

9.52 Assume that the velocity profile in a boundary layer is replaced by a step profile, as shown in the figure, where the velocity is zero adjacent to the surface and equal to the free-stream velocity (U) at a distance greater than δ_* from the surface. Assume also that the density is uniform and equal to the free-stream density (ρ_∞). The distance δ_* (displacement thickness) is so chosen that the mass flux corresponding to the step profile is equal to the mass flux through the actual boundary layer. Derive an integral expression for the displacement thickness as a function of u , U , y , and δ .

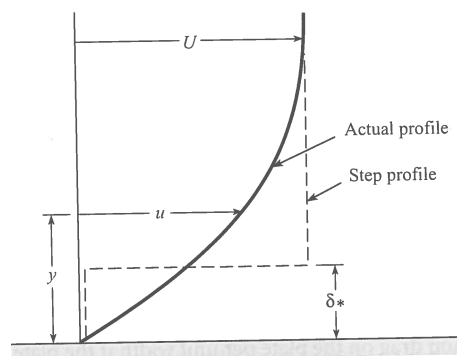
9.53 Because of the reduction of velocity associated with the boundary layer, the streamlines outside the boundary layer are shifted away from the boundary. This amount of displacement of the streamlines is defined as the displacement thickness δ_* . Using the expression developed in Prob. 9.52, evaluate the displacement thickness of the boundary layer at the downstream edge of the plate (point A) in Prob. 9.51.



PROBLEMS 9.51, 9.53

9.54 Use the expression developed in Prob. 9.52 to find the ratio of the displacement thickness to the boundary layer thickness for the turbulent boundary layer profile given by

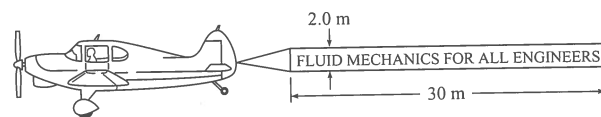
$$\frac{u}{U_0} = \left(\frac{y}{\delta}\right)^{1/7}$$



PROBLEM 9.52

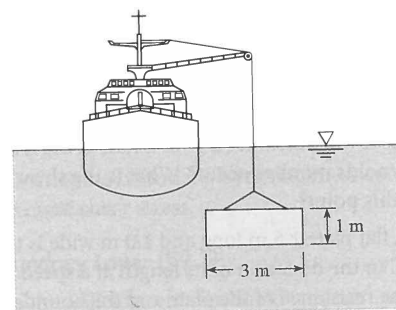
9.55 **PLUS** What is the ratio of the skin friction drag of a plate 30 m long and 5 m wide to that of a plate 10 m long and 5 m wide if both plates are towed lengthwise through water ($T = 20^\circ\text{C}$) at 10 m/s?

9.56 **PLUS** Estimate the power required to pull the sign shown if it is towed at 41 m/s and if it is assumed that the sign has the same resistance characteristics as a flat plate. Assume standard atmospheric pressure and a temperature of 10°C .



PROBLEM 9.56

9.57 **GO** A thin plastic panel (3 mm thick) is lowered from a ship to a construction site on the ocean floor. The plastic panel weighs 300 N in air and is lowered at a rate of 3 m/s. Assuming that the panel remains vertically oriented, calculate the tension in the cable.



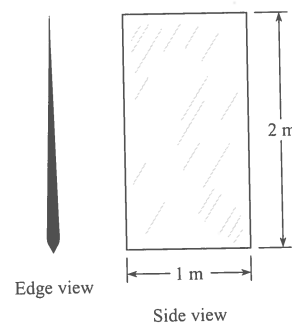
PROBLEM 9.57

9.58 The plate shown in the figure is weighted at the bottom so it will fall stably and steadily in a liquid. The weight of the plate in air is 23.5 N, and the plate has a volume of 0.002 m^3 . Estimate its falling speed in freshwater at 20°C . The boundary layer is normal; that is, it is not tripped at the leading edge.

In this problem, the final falling speed (terminal velocity) occurs when the weight is equal to the sum of the skin friction and buoyancy.

$$W = B + F_s = \gamma V + \frac{1}{2} C_f \rho U_0^2 S$$

Hints: Find the final falling speed. This problem requires an iterative solution.

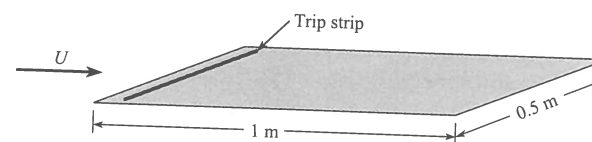


PROBLEM 9.58

9.59 **PLUS** A turbulent boundary layer develops from the leading edge of a flat plate with water at 20°C flowing tangentially past the plate with a free-stream velocity of 7.7 m/s. Determine the thickness of the viscous sublayer, δ' , at a distance 7.8 m downstream from the leading edge.

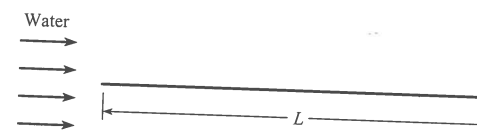
9.60 A model airplane descends in a vertical dive through air at standard conditions (1 atmosphere and 20°C). The majority of the drag is due to skin friction on the wing (like that on a flat plate). The wing has a span of 1 m (tip to tip) and a chord length (leading edge to trailing edge distance) of 10 cm. The leading edge is rough, so the turbulent boundary layer is "tripped." The model weighs 3 N. Determine the speed (in meters per second) at which the model will fall.

9.61 **PLUS** A flat plate is oriented parallel to a 24-m/s airflow at 20°C and atmospheric pressure. The plate is $L = 3\text{ m}$ in the flow direction and 0.5 m wide. On one side of the plate, the boundary layer is tripped at the leading edge, and on the other side there is no tripping device. Find the total drag force on the plate.



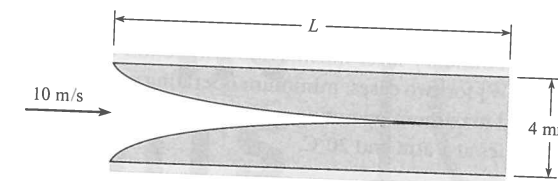
PROBLEM 9.61

9.62 An engineer is designing a horizontal, rectangular conduit that will be part of a system that allows fish to bypass a dam. Inside the conduit, a flow of water at 5°C will be divided into two streams by a flat, rectangular metal plate. Calculate the viscous drag force on this plate, assuming boundary-layer flow with free-stream velocity of 3 m/s and plate dimensions of $L = 1.8\text{ m}$ and $W = 1.2\text{ m}$.



PROBLEM 9.62

9.63 A model is being developed for the entrance region between two flat plates. As shown in the figure, it is assumed that the region is approximated by a turbulent boundary layer originating at the leading edge. The system is designed such that the plates end where the boundary layers merge. The spacing between the plates is 4 mm, and the entrance velocity is 10 m/s. The fluid is water at 20°C . Roughness at the leading edge trips the boundary layers. Find the length L where the boundary layers merge, and find the force per unit depth (into the paper) due to shear stress on both plates.

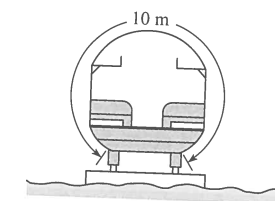


PROBLEM 9.63

9.64 An outboard racing boat "planes" at 110 km/hr over water at 15°C . The part of the hull in contact with the water has an average width of 1 m and a length of 2.5 m. Estimate the power required to overcome its shear force.

9.65 A motor boat pulls a long, smooth, water-soaked log (0.5 m in diameter and 50 m long) at a speed of 1.7 m/s. Assuming total submergence, estimate the force required to overcome the shear force of the log. Assume a water temperature of 10°C and that the boundary layer is tripped at the front of the log.

9.66 **PLUS** High-speed passenger trains are streamlined to reduce shear force. The cross section of a passenger car of one such train is shown. For a train 81 m long, (a) estimate the shear force for a speed of 81.1 km/hr and (b) for one of 204 km/hr. What power is required for just the shear force at these speeds? These two power calculations will be answers (c) and (d) respectively. Assume $T = 10^\circ\text{C}$ and that the boundary layer is tripped at the front of the train.



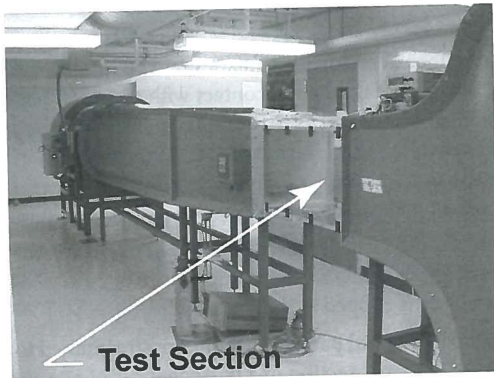
PROBLEM 9.66

9.67 Consider the boundary layer next to the smooth hull of a ship. The ship is cruising at a speed of 15 m/s in 15°C freshwater. Assuming that the boundary layer on the ship hull develops the same as on a flat plate, determine

- The thickness of the boundary layer at a distance $x = 30\text{ m}$ downstream from the bow.
- The velocity of the water at a point in the boundary layer at $x = 30\text{ m}$ and $y/\delta = 0.50$.
- The shear stress, τ_0 , adjacent to the hull at $x = 30\text{ m}$.

9.68 A wind tunnel operates by drawing air through a contraction, passing this air through a test section, and then exhausting the air using a large axial fan. Experimental data are recorded in the test section, which is typically a rectangular section of duct that is made of clear plastic (usually acrylic). In the test section, the velocity should have a very uniform distribution; thus, it is important that the boundary layer be very thin at the end of the test section. For the pictured wind tunnel, the test section is square with a dimension of $W = 457\text{ mm}$

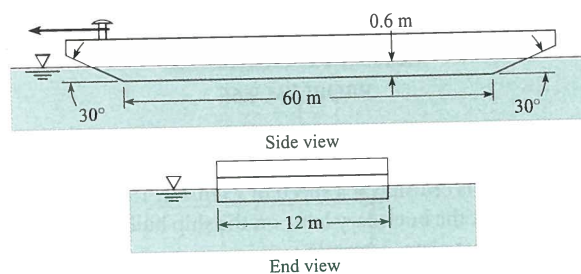
on each side and a length of $L = 914$ mm. Find the ratio of maximum boundary-layer thickness to test section width $[\delta(x = L)/W]$ for two cases: minimum operating velocity (1 m/s) and maximum operating velocity (70 m/s). Assume air properties at 1 atm and 20°C.



PROBLEM 9.68 (Photo by Donald Elger)

9.69 A ship 180 m long steams at a rate of 8 m/s through still freshwater ($T = 10^\circ\text{C}$). If the submerged area of the ship is 4600 m^2 , what is the skin friction drag of this ship?

9.70 A river barge has the dimensions shown. It draws 0.6 m of water when empty. Estimate the skin friction drag of the barge when it is being towed at a speed of 3 m/s through still freshwater at 15°C .



PROBLEM 9.70

9.71 **PLUS** A supertanker has length, breadth, and draught (fully loaded) dimensions of 325 m, 48 m, and 19 m, respectively. In open seas the tanker normally operates at a speed of 9.27 m/s. For these conditions, and assuming that flat-plate boundary-layer conditions are approximated, estimate the skin friction drag of such a ship steaming in 10°C water. What power is required to overcome the skin friction drag? What is the boundary-layer thickness at 300 m from the bow?

9.72 A model test is needed to predict the wave drag on a ship. The ship is 150 m long and operates at 9 m/s in seawater at 10°C . The wetted area of the prototype is 2300 m^2 . The model/prototype scale ratio is 1/100. Modeling is done in freshwater at 15°C to match the Froude number. The viscous drag can be calculated by assuming a flat plate with the wetted area of the model and a length corresponding to the length of the model. A total drag of 0.5 N is measured in model tests. Calculate the wave drag on the actual ship.

9.73 A ship is designed so that it is 250 m long, its beam measures 30 m, and its draft is 12 m. The surface area of the ship below the water line is 8800 m^2 . A 1/40 scale model of the ship is tested and is found to have a total drag of 26.0 N when towed at a speed of 1.45 m/s. Using the methods outlined in Section 8.9, answer the following questions, assuming that model tests are made in freshwater (20°C) and that prototype conditions are seawater (10°C).

- To what speed in the prototype does the 1.45 m/s correspond?
- What are the model skin friction drag and wave drag?
- What would the ship drag be in saltwater corresponding to the model test conditions in freshwater?

9.74 A hydroplane 3 m long skims across a very calm lake ($T = 20^\circ\text{C}$) at a speed of 15 m/s. For this condition, what will be the minimum shear stress along the smooth bottom?

9.75 Estimate the power required to overcome the shear force of a water skier if he or she is towed at 50 km/hr and each ski is 1.2 m by 15 cm. Assume the water temperature is 15°C .

9.76 If the wetted area of an 80-m ship is 1500 m^2 , approximately how great is the surface drag when the ship is traveling at a speed of 15 m/s. What is the thickness of the boundary layer at the stern? Assume seawater at $T = 10^\circ\text{C}$.

FLOW IN CONDUITS 10

Chapter Road Map

This chapter explains how to analyze flow in conduits. The primary tool, the energy equation, was presented in Chapter 7. This chapter expands on this knowledge by describing how to calculate head loss. In addition, this chapter explains how to design pumps into systems and how to analyze a network of pipes.

Learning Objectives

STUDENTS WILL BE ABLE TO

- Define a conduit. Classify a flow as laminar or turbulent. Define or calculate the Reynolds number. (§10.1)
- Describe developing flow and fully developed flow. Classify a flow into these categories. (§10.1)
- Specify a pipe size using Diameter Normal (DN). (§10.2)
- Describe total head loss, pipe head loss, and component head loss. (§10.3)
- Define the friction factor f . List the steps to derive the Darcy-Weisbach equation. (§10.3)
- Describe the physics of the Darcy-Weisbach equation and the meaning of the variables that appear in the equation. Apply this equation. (§10.3)
- Calculate h_f or f for laminar flow. (§10.5)
- Describe the main features of the Moody diagram. Calculate f for turbulent flow using the Moody diagram or the Swamee-Jain correlation. (§10.6)
- Solve turbulent flow problems when the equations cannot be solved by algebra alone. (§10.7)
- Define the minor loss coefficient. Describe and apply the combined head loss equation. (§10.8)
- Define hydraulic diameter and hydraulic radius and solve relevant problems. (§10.9)
- Solve problems that involve pumps and pipe networks. (§10.10)

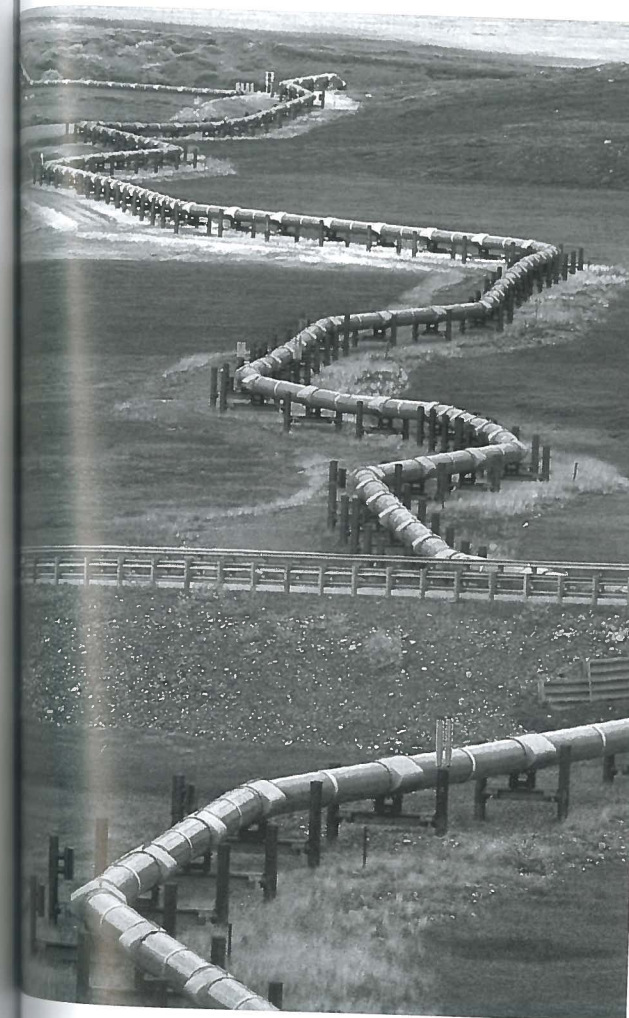


FIGURE 10.1

The Alaskan pipeline, a significant accomplishment of the engineering profession, transports oil 1286 km across the state of Alaska. The pipe diameter is 1.2 m, and 44 pumps are used to drive the flow. This chapter presents information for designing systems involving pipes, pumps, and turbines. (© Eastcott/Momatiuk/The Image Works.)